Systems of Equations

Finite Math

6 March 2017

Quiz

How was your weekend?

Motivating Example

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Package 1: 2 adult tickets and 1 child ticket for \$32
Package 2: 1 adult ticket and 3 child tickets for \$36

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Based off of this information, can we figure how much the adult and child ticket discount prices are?

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This is a system of two linear equations in two variables. To find the answer, we need to find a pair of numbers (A, C) which satisfy *both* equations simultaneously.

Definition

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Given the linear system

$$\begin{array}{rcrcl} ax & + & by & = & h \\ cx & + & dy & = & k \end{array}$$

where a, b, c, d, h, and k are real constants, a pair of numbers $x = x_0$ *and* $y = y_0$ *(often written as an ordered pair* (x_0, y_0)*) is a solution of this system if each equation is satisfied by the pair. The set of all such ordered pairs is called the solution set for the system. To solve a system is to find its solution set.*

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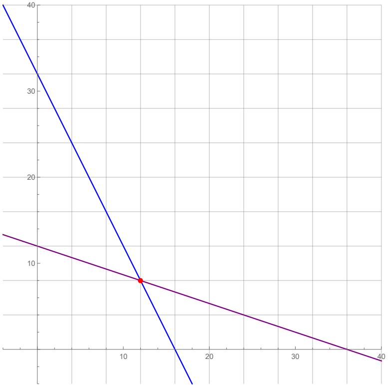
There are a few ways we can go about solving this: *graphically*, using *substitution*, and *elimination by addition*.

Solving by Graphing

To solve this problem by graphing, we simply graph the two equations in the system, then find the intersection. Since we're relying on a graph to find this point, we need to check our solution in the equations of the system.



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This verifies the solution.

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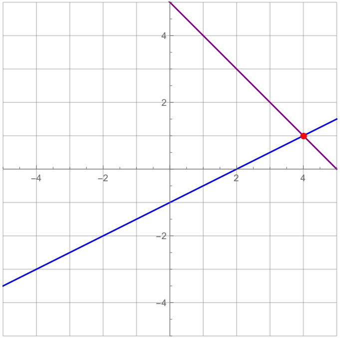

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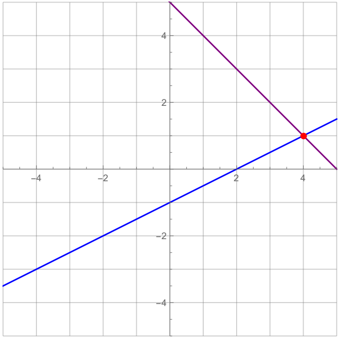

In this case, like before, we see only the *one solution* at (4, 1). (You should check this in the system!)

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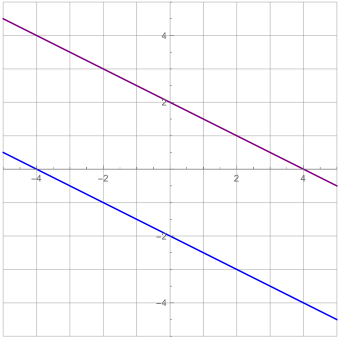

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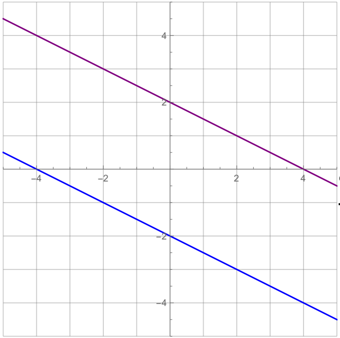

In this case, the lines are parallel and so they never intersect. In this case, there is *no solution*.

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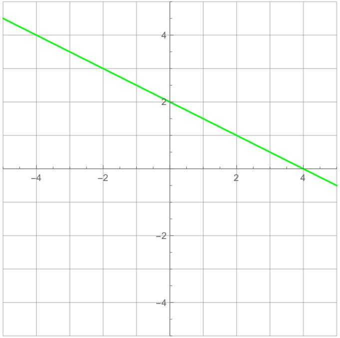

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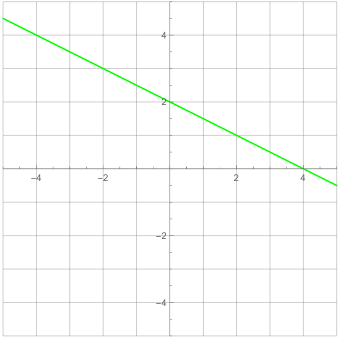

Here, both of the lines are exactly the same.
In this case, there is an infinite number of solutions.

Terminology

Definition

A system of linear equations is called consistent if it has one or more solutions and inconsistent if it has no solutions. Further, a consistent system is called independent if it has exactly one solution (called the unique solution) and is called dependent if it has more than one solution. Two systems of equations are called equivalent if they have the same solution set.

Theorem

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The linear system

$$\begin{array}{rcrcl} ax & + & by & = & h \\ cx & + & dy & = & k \end{array}$$

must have

1. *Exactly one solution (consistentent and independent).*
2. *No solution (inconsistent).*
3. *Infinitely many solutions (consistent and dependent).*

There are no other possibilities.

Now You Try It!

Example

Solve the following systems of equations using the graphing method. Determine whether there is one solution, no solutions, or infinitely many solutions. If there is one solution, give the solution.

(a)

$$\begin{array}{rcrcr} x & + & y & = & 4 \\ 2x & - & y & = & 5 \end{array}$$

(b)

$$\begin{array}{rcrcr} 6x & - & 3y & = & 9 \\ 2x & - & y & = & 3 \end{array}$$

(c)

$$\begin{array}{rcrcr} 2x & - & y & = & 4 \\ 6x & - & 3y & = & -18 \end{array}$$

Solving by Substitution

When solving a system by substitution, we solve for one of the variables in one of the equations, then plug that variable into the other equation.

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Example

Solve the following system using substitution

$$\begin{array}{rcrcl} 2x & - & y & = & 3 \\ x & + & 2y & = & 14 \end{array}$$

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Solve the following system using substitution

$$\begin{aligned} 3x &+ 2y &= -2 \\ 2x &- y &= -6 \end{aligned}$$

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Solve the following system using substitution

$$\begin{array}{rcrcr} 3x & + & 2y & = & -2 \\ 2x & - & y & = & -6 \end{array}$$

Solution

$x = -2, y = 2$

Solving Using Elimination

We now turn to a method that, unlike graphing and substitution, is generalizable to systems with more than two variables easily. There are a set of rules to follow when doing this

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Theorem

A system of linear equations is transformed into an equivalent system if

1. *two equations are interchanged*
2. *an equation is multiplied by a nonzero constant*
3. *a constant multiple of one equation is added to another equation.*

Solving Using Elimination

Example

Solve the following system using elimination

$$\begin{array}{rcrcr} 3x & - & 2y & = & 8 \\ 2x & + & 5y & = & -1 \end{array}$$

Now You Try It!

Example

Solve the system using elimination

$$\begin{aligned} 5x - 2y &= 12 \\ 2x + 3y &= 1 \end{aligned}$$

Now You Try It!

Example

Solve the system using elimination

$$\begin{array}{rcrcl} 5x & - & 2y & = & 12 \\ 2x & + & 3y & = & 1 \end{array}$$

Solution

$x = 2, y = -1$